Vague Objects and the Logic of Vagueness

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Abstract

This paper is an inquiry into the problems of conceptual and ontological vagueness. The main emphasis is on the problem of vague objects and the related issue of vague identity, and my approach is to use the framework and semantics of quantified modal logic to throw light on these questions.

1 Introduction

It would be wrong to say that the phenomenon of vagueness is a neglected subject in modern philosophy[1], but it is also an exaggeration to say that it is "one of the hottest topics in the philosophy of language and the philosophy of logic"[2] today. What I shall say is that vagueness deserves much more attention than it usually gets. The problem of vagueness is more than just a logical puzzle; it has in fact far-reaching consequences for our view of human knowledge and of the reality it is meant to be knowledge of. Besides, as I shall argue, an awareness of the significance of the problem of vagueness that seems to have been widespread among early Greek philosophers has had a lasting and deep although little noticed influence not only on the philosophical tradition from antiquity, but on the development of scientific thought as well.

[1] Over the last decades a considerable number of articles have been dedicated to this subject, and at least one comprehensive book: Timothy Williamson's *Vagueness* [13]. This is a thorough and instructive discussion of the whole field of vagueness problems, and in the following I shall refer to it repeatedly. I agree with Williamson on most points, and there is only one point in his book I take exception to: his treatment of vague identity and unclarity *de re*.

[2] This is Roy Sorensen's claim, quoted on the cover of Williamson's *Vagueness*.

That language is often vague is a trivial truth. That vagueness is inherent in every language to a greater or smaller degree is not so trivial, but nonetheless true, as we shall see. Now, to the extent that language is essentially vague, thought must be vague too, it seems. Not only our words, but also our concepts are vague then.

A conceptual scheme can be compared to a coordinate system. Like a map superimposes its coordinate system on the landscape, a theory superimposes its conceptual scheme on reality, carving the world up in manageable pieces according to a classification. So if a conceptual scheme is vague, it seems to follow that it carves the world up in pieces that are somewhat lacking in definition, with blurred outlines. The question then arises whether the objects of reality may themselves be vague in the sense of being underdefined. After all, what other than some classification system can divide the world into objects; and if every conceptual scheme is vague, how can there ever be a precise definition of things?

I hope what I have said so far will suffice to give a sketchy overview of the field we are going to enter. In the following I shall first present a few examples of problematic vagueness, two of them old and dignified philosophical puzzles, before I go on to discuss some of the vagueness problems, including the problem of 'slippery slope' *sorites* reasoning. The main emphasis in this paper will be on the question of vague objects and the related problem of vague identity, however; and my approach is to use the framework and semantics of quantified modal logic to throw light on these issues.

2 Problems of Vagueness

Eubulides of Miletus, a contemporary of Aristotle, is known for a number of puzzles, among them the sorites (the heap-problem)[3]: How many grains make up a heap? This question turns on the vagueness of the term "heap". The suggested implication is that because of the vagueness one grain more or less should not make any difference: if n grains are too few to make up a heap, then $n + 1$ are not enough either; and if n grains do make up a heap, then we still have a heap when one grain is removed. But this leads to absurdity. Starting with one grain, which is clearly not a heap, we conclude falsely, by a chain of modus ponens inferences, that, say, 10000 grains do not make up a heap; or starting with 10000 grains, which clearly make up

[3] [13, p. 8ff.].

a heap, we conclude, correspondingly, that the heap will remain even if all the grains are removed.

The fallacy of this kind of slippery slope reasoning is well illustrated in a Norwegian folk tale: One winter day a woman pulls her sleigh into the forest in order to collect firewood. She gathers dry branches and twigs and loads them on her sleigh. For each twig she puts on top of the load she says to herself, "If I can pull this, then I can pull this too." In the end she cannot move the sleigh. So she starts the process of unloading, saying to herself for each twig she removes, "If I can't pull this, then I can't pull this either," and ends up pulling her empty sleigh back home.

That her reasoning is fallacious is easy to see, and the source of the fallacy is equally easy to detect in her case: She has failed to realize that there is a method of deciding accurately, at any point, whether she will be able to pull the load or not, a practical criterion—she just has to try. The heap is a different matter in this respect, since we have no clear and simple criteria for deciding exactly how large a collection of grains must be to count as a heap. This means that while there are certain quantities of grains that obviously count as heaps on the one hand, and other, smaller quantities that obviously don't on the other hand, there are also borderline cases in which it is neither obvious that the grain collection makes up a heap nor that it doesn't. In such cases we may either disagree between us on what to decide, or we may be unable to reach a decision at all.

The problem of the heap can be considered the basic problem of vagueness. The problem of Theseus' ship, popular among the sophists, is slightly different, turning on identity[4]: Theseus was the Athenian prince who, according to the myth, killed the Minotaur of Knossos and escaped from the labyrinth thanks to the thread the Cretan princess Ariadne gave him. He returned to Athens, and his ship was kept after that by the Athenians and used for a ceremonial voyage to Delos once a year at least until the death of Socrates more than 800 years later[5]. Over all these years the ship must have been repaired again and again, and each time one or more of its constituent boards must have been replaced. We can safely conclude that all of the original boards had been replaced after 800 years. But was it then still the same

[4] See [15].

[5] Plato's *Crito* starts with Crito informing the imprisoned Socrates early in the morning that the ship from Delos will return later on the same day, urging him to flee at once. While the ship was on its ceremonial voyage, no prisoners were executed in Athens, and that was the reason Socrates had to stay in prison for almost a month before he took the poison.

ship? The problem is, how many boards of a ship can be replaced before the original ship is gone and a new ship has come into existence? Replacing the boards of a ship one by one is similar to removing the grains that constitute a heap one after the other: when does a substantial change take place? On the one hand, replacing one board should not affect the identity of the ship. On the other hand, if all the boards were replaced at once, that is, if we discarded the old ship and built a new one, it would obviously be a different ship. What then if we replace all the boards of a ship one by one over a long period of time—is it still the same ship? There is no obvious answear to that question. The concept of a ship is vague in the sense that it has no clear identity criteria.

The concept of a heap is vague in the respect that it is not clear what counts as a heap. The concept of a ship is vague in the respect that it is not clear what counts as the same ship. The concept of a mountain is vague in both these respects: It is not clear what counts as a mountain and what is just a hill, and it is not clear what counts as one mountain and what counts as two or more, and hence not clear what counts as the same mountain: When do we have one mountain with two peaks, and when do we have two different mountains with a small valley between them? There are no clear criteria for deciding such questions. Our unability to give a precise answer to the last question has serious consequences. First, it means that we have no criteria for deciding how many different mountains there are in the world. Second, it also shows that the concept of a mountain must be vague in an additional respect: It is not clear where exactly in the landscape the boundaries of a mountain are. Where does the mountain end and the valley begin? Where is the end of one mountain and where is the beginning of another? We lack criteria for deciding this, too.

Now, if not before, we will perhaps be tempted to think that not only concepts, but also the objects they are concepts of may be vague. For the boundaries of a mountain are not anything like a physical cleavage between the mountain and its environment, but a more or less arbitrary drawn imaginary line; and if it is not clear where exactly this line is drawn, must not that mean that the outline of the topographical structure we call a mountain is fuzzy, and that the mountain itself is an underdefined, vague entity?

A discussion of this problem must wait. Now I want to generalize what I have just said about mountains. It is true not only of the concept of a mountain, but of our concepts of spatial objects in general that our criteria are defective when it comes to deciding where the spatial boundaries of these objects are. Unlike a mountain, a table may give the appearance of being a well-defined spatial structure, but that is only so because our vision isn't better than it is. If we could see the molecules that constitute the table, we would also see that there is a problem of deciding which molecules at the surface of the table are parts of the table and which are not.[6]

If there are vague objects then we are certainly among them. The spatial boundaries of our bodies are as fuzzy as those of all spatial objects, or more fuzzy, in fact, because we are living organisms: are, for instance, the contents of my stomach a genuine part of me? Our temporal boundaries are fuzzy as well: Exactly when do we come into being? Exactly when do we pass away? These questions are of great practical importance, and give rise to heated debates, but even so it seems almost impossible to give precise, well-founded and convincing answers to them. There are no criteria that everybody can agree on, so the debate goes on.

Also our concepts of abstract entities are vague to a certain degree—vague in the sense that certain questions about such entities seem to be undecidable because of a lack of criteria. Number theory is apparently the very paradigm of exactness, but even so it turns out that the ontological status of natural numbers is unclear. Are natural numbers classes? If so, what kind of classes? Is Zermelo's conception of each natural number as the unit class of the preceding number, starting with 0 as the empty class, the correct one? Or is Zermelo wrong and Frege right—is the natural number n the class of all classes with n members? How can we decide? We have no criteria. So the concept of natural numbers is vague as concerns the real nature of these abstract entities.[7] Are numbers as such vague and underdefined, then; or is it just vague what numerals refer to? In general: are objects vague, or is it just vague which objects we talk about? This is a question we shall return to.

3 Flight from the Horror of Vagueness: The Origins of Scientific Thought

Now I want to digress a little, and take a closer look at the problem of vagueness in the history of philosophy and science. It is my contention that this problem has played a more important role as a motivational force behind the development of scientific thought than is usually realized. What is well known is that the problem of change was one of the predominant themes in

[6] I have borrowed this example from Quine [11].

[7] This example is also borrowed from Quine [10].

ancient Greek philosophy; the Greek philosophers, including both Plato and Aristotle, looked at the changeable objects of the senses with suspicion, assuming that no real knowledge could be had of them as such. But why? My answer is that they thought that these objects had to be vague. It was the horror of vagueness that drove Parmenides, Democritus and Plato to draw their somewhat different distinctions between appearance and reality, a type of distinction that still underlies modern scientific thought. The problem of change is: how can something remain the same when it goes through change? This is really a problem of vagueness, it stems from the fact that our identity criteria for changeable things are not clear. Because this problem seems to invite contradiction—a thing both remains and does not remain the same when it goes through change—prominent Greek philosophers, starting with Parmenides, postulated a stable and unchanging reality beyond the flimsy and fuzzy appearance we, as they saw it, all too often mistake for the real thing.

"Never say about that which is not that it is!" Parmenides admonished. According to him, what is, the ultimate reality, is the One—eternal, unchanging, free of all differences and diversity, the same in every respect. The world of appearance is nonbeing, a mere illusion. The evident weakness of Parmenides's bold theory is, however, that it cannot account for this illusion. So those who came after him tried to revise the theory so that it could explain appearance as an effect of reality. There were at least two ways of doing that, one of them is Plato's, the other that of Democritus. Plato's theory of the Forms multiplies form, starting with the One as the form of forms[8]. Democritus saw Parmenides's One as a material unit, and postulated instead of the one One an infinitude of similar indivisible material units moving through empty space and interacting according to simple mechanical laws. Democritus's resulting Atom theory held great promise of being able to explain the world of appearance: Changeable things like we are ourselves are just temporary aggregates of atoms; the qualities of the senses are just the effect of the impact of the essentially soundless and colorless reality around us upon our constituent soul atoms, which start to vibrate like the strings of a harp.

Through Epicurus, who adopted and refined Democritus's thought, and then Lucretius, who in his famous *De Rerum Natura* exposed the teaching of Epicurus in Latin, the Atom theory came to have a decisive influence on the development of the physical sciences in the 17. and 18. centuries. That

[8]See in particular Plato's *Parmenides*.

especially Galileo was deeply influenced by Lucretius is easy to see from his choice of problems. The other main influence on the new sciences, accentuating the importance of a mathematical theory language, was—interesting enough—that of Plato and the Pythagoreans.

4 Vagueness and the Principle of Bivalence

It is perhaps only natural to think that an assertion must lack a definite truth value if the criteria for deciding its truth or falsity are missing because one or more of its constituent terms are vague. At least, many have made the assumption that we must give up the principle of bivalence in order to give an adequate account of vagueness. However, as we shall see now, this is not the case. It is not necessary, and it will not even help. On the contrary, it will actually prevent a solution to the problem of sorites reasoning. I am saying this in spite of the fact that most approaches to vague statements are based on some variety of many-valued logic (ranging from three-valued to continuum-valued). This may be fine as an approximation to vagueness for certain practical purposes (although many-valued logics have some hard problems of their own), but it does not and cannot solve the basic vagueness problem, illustrated by the heap.[9]

Imagine that we are shown different quantities of grains, in increasing or decreasing order, and for each of them are asked the question, Is this a heap? We are to answer Yes or No, and to say nothing if we cannot decide. Starting from one end, one grain is clearly not a heap, and as grain is added to grain one by one our answer will remain for a long time an unhesitating No to the question that is asked. Starting from the opposite end, 10000 grains clearly make up a heap, and our answer will remain an unhesitating Yes for a long time as grain after grain is removed and the same question is again asked each time.

That the concept of a heap is vague is manifested in the fact that there is no sharp boundary between Yes and No; instead there is a kind of twilight zone between the day of clearly Yes and the night of clearly No, and this is also the reason many people think that the assertion, "This is a heap," has no definite truth value in the borderline cases. But if this were so, we should expect the boundaries of the borderline zone to be sharp, that is, that there should be a sharp dividing line between clearly Yes and undecidable and an equally sharp dividing line between undecidable and clearly No. The fact is

[9]See [13, ch. 4–5].

that the opposite holds, that the second-level boundaries of the borderline zone are as fuzzy as the first-level boundary between Yes and No. When we approach the twilight zone from the day side, our Yes, that started out clear and loud, will come out more and more hesitatingly until we don't know what to say anymore and silence sets in; also some of us will start to hesitate or fall silent before others. Correspondingly, when we move from the night side into the twilight zone our No will fade out only gradually and almost unnoticeable, and again we will not all keep the same pace. A hesitating Yes or No has little value as evidence for or against the hypothesis that the quantity of grains makes up a heap, then; and silence is compatible with the truth as well as with the falsity of that hypothesis.

What this means is that the assertion "This is a heap" can be true even though it is not clear that it is true, or false even though it is not clear that it is false; something can be vague and true or vague and false at the same time, so vagueness does not, at least not invariably, take on some intermediate value between true and false.[10] Now I want to elaborate on this by discussing the problem of sorites reasoning in more detail[11]. I am going to show that the way to a solution of the sorites problem does not go through abandonment of the principle of bivalence.

The expression "it is clear that" (or "clearly") can be regarded as a sentence operator. What I have just argued, on the evidence of the nature of vagueness, is, in effect, that this operator is not redundant: Though "This is a heap" follows from "It is clear that this is a heap", the converse inference is not valid; on some occasions the sentence "This is a heap" may be true and the sentence "It is clear that this is a heap" false. On the view that a vague assertion always lacks a definite truth value, the clarity operator ought to be redundant, however; what is the case ought always to be clearly the case. But the nature of vagueness speaks against this view. Given that the clarity operator is not redundant, one should expect that it can be meaningfully iterated, and that iteration of this operation can be combined with negation in different ways as well, to express different higher-order vagueness and clarity concepts. Here is an example of a problem that brings in higher-order vagueness: Can it be unclear whether something is the case without its being clear that it is unclear? This seems at least to be a meaningful question, whatever the answer will be. (I think the answer should be Yes.) And here is a parallel example of a problem of second-order clarity: It is an

[10] I first made this point in a paper in Norwegian, *Vage objekter?* [2].

[11] I am indebted to Williamson [13] for several points in the following discussion.

interesting question whether it may sometimes be clear that p even though it is not clear that it is clear that p. Granted that this question makes sense in the first place, the answer we give to it will actually be decisive for our ability to solve the sorites puzzle, as we shall see shortly.

The crucial assumption of the sorites reasoning is the following:

S+ If n grains do not make up a heap then $n+1$ grains do not make up a heap,

or, conversely and equivalently:

S– If n grains make up a heap then $n-1$ grains make up a heap.

(I refer to the assumption as "S" for "sorites", and distinguish between the two equivalent formulations in an obvious way as "S+" and "S–" respectively.) To make the assumption S leads to disaster as we have seen. By a chain of modus ponens inferences we are then forced to conclude, dependent on the premise we start with, that the grains we assemble will never add up to a heap, no matter how many they are, or that the heap will remain a heap forever, no matter how many grains we subtract from it. So S must be false, however compelling it seems to be, and I think the reason why it appears to be so convincing in spite of leading to absurdity, is that it comes very close to a truth. So what is this truth, what should S be replaced with? In my opinion, what is wrong with S is that this assumption fails to take clarity into account: It is not that one grain more or less makes no difference; because it does—we see that when the grains add up. It is that one grain more or less doesn't *clearly* make a difference. The truth (or rather truths, because there are two of them, twin principles involving the clarity operator) which I think should replace the unfortunate false assumption S, is therefore the following:

T1+ If n grains do not make up a heap then it is not clear that $n+1$ grains make up a heap,

or, conversely and equivalently:

T1– If it is clear that n grains make up a heap then $n-1$ grains make up a heap,

and

T2– If n grains make up a heap then it is not clear that $n-1$ grains do not make up a heap, or, again conversely and equivalently:

T2+ If it is clear that n grains do not make up a heap then $n+1$ grains do not make up a heap.

(I refer to the pair of twin principles as "T" for "truth", and I distinguish between the two parts as "T1" and "T2" respectively, and between the two equivalent formulations of each by using the plus and minus sign.)[12]

T seems to capture our intuitions about vagueness much better than S. And replacing S with T also breaks the modus ponens chain: From the premise that it is clear that i grains make up a heap, and the additional premise, from T1–, that if it is clear that i grains make up a heap then $i-1$ grains make up a heap, we can only conclude that $i-1$ grains make up a heap, not that it is clear that $i-1$ grains make up a heap, which is what we would need as a new premise for the next modus ponens inference.[13] Even if we assume that it is clear that i grains clearly make up a heap, and, as we should (because this is a principle), that it is clear that if i grains clearly make up a heap then $i-1$ grains make up a heap, we can only conclude that it is clear that $i-1$ grains make up a heap, but—unless we assume that if it is clear that p then it is clear that it is clear that p—not that it is clear that $i-1$ grains clearly make up a heap, which we would need as a new premise in order to repeat the inference one step down.

For this solution to the problem of sorites reasoning to work, it is, first, of crucial importance that the clarity operator be nonredundant, which as a consequence permits us to assume that it can be meaningfully iterated as well, allowing for higher-order clarity and vagueness, and then, second, it is of equally decisive importance that the assumption that if it is clear that p then it is clear that it is clear that p be false, for if that were to be used as a principle of reasoning we would obviously be on the slippery slope again. It seems very reasonable to reject this assumption, however, given the nature of vagueness and clarity.

If the view that vague assertions lack a definite truth value were right, and the principle of bivalence had to be abandoned, the clarity operator

[12] Williamson [13, §8.2] discusses a parallel principle with knowledge instead of clarity on the assumption that vagueness is a kind of ignorance.

[13] And the other way round: From the premise that i grains do not make up a heap, and the additional premise, from T1+, that if i grains do not make up a heap then it is not clear that $i+1$ grains make up a heap, we can only conclude that it is not clear that $i+1$ grains make up a heap, not that $i+1$ grains do not make up a heap.—Since reasoning with T2 is parallel to reasoning with T1, I see no point in giving a separate example of that.

would be redundant. The principle T would thus be conflated with the disastrous assumption S, and the absurd conclusions of sorites reasoning would then follow. So in order to keep the hope of being able to give an adequate account of vagueness alive, we ought to keep to the principle of bivalence as long as we can.

5 The Logic of Vagueness and Clarity

I shall follow Evans[14] and use the symbols "$\triangle$" and "∇" as short for "it is clear that" and "it is unclear whether" respectively, so that "$\triangle p$" is to be read as "It is clear that p", clear in the sense that it is a fact and not vague, and "∇p" is to be read as "It is unclear whether p", unclear in the sense that it is vague.

Each of the sentence operators "$\triangle$" and "∇" can be defined by, and thus reduced to, the other. "$\triangle p$" can be defined as "$p \,\&\, {\sim}\nabla p$"; "∇p" can be defined as "${\sim}\triangle p \,\&\, {\sim}\triangle{\sim}p$". It doesn't really matter which operator we take as primitive, but that makes the clarity operator the obvious choice, since things will become a bit simpler this way.

The idea of a logic of vagueness may appear a very weird notion indeed, but now we can see that a logic of vagueness will also be a logic of clarity, and that doesn't sound nearly as strange. This idea is no novelty; the affinity between the logic of vagueness and clarity on one hand and modal logic on the other has been noticed long ago. The parallel in logical behavior between the clarity operator, "$\triangle$", and the necessity operator, "$\Box$", of modal logic is striking, though the vagueness operator, "∇", cannot be compared to the possibility operator, "$\Diamond$". So it then seems that model-theoretic semantics for modal logic, accommodated, should also be adequate for the logic of clarity.

A *model structure*[15] for a language $\mathcal{L}$ that in this case contains the clarity operator has three constituents: (1) a set, $\mathcal{K}$, of possible worlds—or as I prefer to say, and especially in connection with the logic of vagueness and clarity: *draft worlds*—(2) a draft world, $\mathcal{G}$, which is an element of $\mathcal{K}$ with a privileged status—intuitively it coincides with the real world—and (3) a relation, R, on $\mathcal{K}$. Here "$\mathcal{H}_1 R \mathcal{H}_2$" is to have the meaning that the world $\mathcal{H}_2$ is indiscernible in the world $\mathcal{H}_1$ from the world $\mathcal{H}_1$ itself because of the

[14] In [3] he introduces symbols similar to mine in a similar sense.

[15] I shall use the terminology and conceptual apparatus of Kripke [6] in my exposition of model-theoretic semantics.

vagueness of $\mathcal{L}$. Let me explain the import of this: If you are in the world $\mathcal{H}_1$, that is, if this is the real world $\mathcal{G}$, you will not be able to decide because of a lack of criteria that you are not in the world $\mathcal{H}_2$, that is, that $\mathcal{H}_2$ is not the real world. Let me use an oversimplified illustration: Say that if the collection of grains in front of you is a heap, then you are in draft world $\mathcal{H}_1$, which in that case coincides with the real world, because it is true in $\mathcal{H}_1$ that the collection of grains is a heap. But if the collection of grains is not a heap, then you are in draft world $\mathcal{H}_2$, which is then the real world $\mathcal{G}$, because in $\mathcal{H}_2$ it is false that the collection of grains in front of you is a heap. As a matter of (stipulated) fact, there is a heap in front of you, so you are in $\mathcal{H}_1$. But you are not able to decide, because of the vagueness of the term "heap", whether the collection of grains is a heap or not, so you cannot tell which world you are in. This is the intuitive meaning of the indiscernibility relation R. As we shall see soon, to lay down axioms for a logic of clarity is also to define this relation by laying constraints on it.

A *model* of a language is a complete interpretation of it, fixing the meaning of each of its components on a certain level (the level of propositional logic or the level of predicate logic). Given a model structure, a model of a language of propositional logic with the clarity operator, $\mathcal{L}$, consists in a simultaneous value assignment to all the sentences of $\mathcal{L}$. It is a function, ϕ, that basically assigns a truth value to every atomic sentence in $\mathcal{L}$ for each draft world $\mathcal{H}$ in $\mathcal{K}$, and the truth values of composite sentences are then fixed in accordance with a set of rules. A model of a language of predicate logic will be more complex, of course; we will go into that later. But by now we only have to take negation, conjunction and the clarity operation into consideration. We have the following obvious rules for negation and conjunction (where α and β are arbitrary sentences, "$\mathcal{H}$" etc. are variables ranging over the elements of $\mathcal{K}$, and "$\top$" designates the value true, "$\bot$" the value false):

(R$\sim$) For every $\mathcal{H}$, $\phi({\sim}\alpha, \mathcal{H}) = \top$ if and only if $\phi(\alpha, \mathcal{H}) = \bot$,

(R&) For every $\mathcal{H}$, $\phi(\alpha \,\&\, \beta, \mathcal{H}) = \top$ if and only if both $\phi(\alpha, \mathcal{H}) = \top$ and $\phi(\beta, \mathcal{H}) = \top$,

and for the clarity operator the rule is:

(R$\triangle$) For every $\mathcal{H}$, $\phi(\triangle\alpha, \mathcal{H}) = \top$ if and only if $\phi(\alpha, \mathcal{H}') = \top$ for every $\mathcal{H}'$ such that $\mathcal{H} R \mathcal{H}'$.

This completes the description of a model of a propositional logic language with the clarity operator. Since "∇" can be defined by "$\triangle$", the rule for the vagueness operator is derivable from the three rules already laid down, but I shall state it here anyway:

(R∇) For every $\mathcal{H}$, $\phi(\nabla\alpha, \mathcal{H}) = \top$ if and only if $\phi(\alpha, \mathcal{H}') = \bot$ for some $\mathcal{H}'$ such that $\mathcal{H} R \mathcal{H}'$ and $\phi({\sim}\alpha, \mathcal{H}'') = \bot$ for some $\mathcal{H}''$ such that $\mathcal{H} R \mathcal{H}''$

Given a model, for a sentence in $\mathcal{L}$ to be true, plainly and simply, is to be true in $\mathcal{G}$, the draft world in $\mathcal{K}$ that coincides with the real world. A sentence is said to be *valid* if and only if it is true in all models.

Now it follows from (R$\triangle$) that the sentence "It is clear that p" is true if and only if p is true in every draft world that is indiscernible in the real world from the real world itself, and it follows from (R∇) and (R$\sim$) that the sentence "It is unclear whether p" is true if and only if p is true in some draft world that is indiscernible in the real world from the real world itself, and false in some other draft world the real world bears this indiscernibility relation to.[16] Our model structure allows for higher-order clarity and vagueness. For example, the sentence "It is clear that it is clear that p" is true if and only if "It is clear that p" is true in every draft world $\mathcal{H}$ that is indiscernible in the real world from the real world itself, and "It is clear that p" is true in $\mathcal{H}$ if and only if p is true in every draft world $\mathcal{H}'$ that is indiscernible in $\mathcal{H}$ from $\mathcal{H}$ itself.

We still have to decide, however, by laying down axioms and rules of inference for the clarity operator, which sentences of $\mathcal{L}$ shall be considered valid (presupposing, of course, the axioms and rules of inference of standard propositional logic). So far only the intuitive meaning of the indiscernibility relation, R, has been given, and no constraints have been laid on it. Now the time has come to discuss what the properties of this relation should be. Should it, for instance, be reflexive, should it be symmetric, and should it be transitive?

First, R should obviously be reflexive. Every world must be indiscernible from itself in itself. The condition on the clarity operation corresponding to

[16] There is a certain similarity between the model-theoretic approach to the logic of clarity and the application of the method of supervaluations to the problem of vagueness: Supervaluationism treats vagueness as a kind of ambiguity. A supervaluation of a statement is a precisification of it that is either clearly true or clearly false. If all supervaluations of a statement are true, then the statement itself is *supertrue*; if all supervaluations of it are false, then it is *superfalse*. Supervaluationism claims that truth is supertruth and falsity superfalsity. Vague statements, being true on some supervaluations and false on others, are not simply true or false, then; so in spite of acknowledging the problem of higher-order vagueness, supervaluationism has to give up bivalence. See [13, ch. 5] for an exposition of this approach and a discussion of the difficulties it leads to.

the reflexivity of R is that for any sentence, p, if it is clear that p then p. "It is clear that p" is true if and only if p is true in every world the real world bears the relation R to, so given that the real world bears this relation to itself, p must then be true. Every instance of the schema "$\triangle\alpha \supset \alpha$" is valid in a model structure with reflexive R, so here we have a good candidate for an axiom:

(AR) $\vdash \triangle\alpha \supset \alpha$.

(I refer to it as "AR" for "the Axiom corresponding to Reflexivity". The symbol "$\vdash$" is a validity label.)

Second, should R be symmetric? Even though it would seem very natural to make this assumption, it is not entirely obvious, and because of that I have taken great care to say that the meaning of "$\mathcal{H}_1 R \mathcal{H}_2$" is that the world $\mathcal{H}_2$ is indiscernible *in* the world $\mathcal{H}_1$ from the world $\mathcal{H}_1$ itself—not just that $\mathcal{H}_2$ is indiscernible from $\mathcal{H}_1$. It is a possibility we should not ignore that $\mathcal{H}_1$ may be discernible from $\mathcal{H}_2$ in $\mathcal{H}_2$ even though the two worlds are indiscernible in $\mathcal{H}_1$. The condition on the clarity operation that would correspond to the symmetry of R is that for any sentence, p, if p then it is clear that it is not clear that not p. So symmetry of R would make every instance of

$$\alpha \supset \triangle{\sim}\triangle{\sim}\alpha$$

(the counterpart of Brouwer's axiom) valid. But since R cannot be assumed with certainty to be symmetric, an axiom corresponding to symmetry seems not be be justified here. We shall return to this question later, however, in a discussion of a problem with vague objects and quantification into vagueness contexts.[17]

Third, R should obviously not be transitive. There is no reason to assume that if the world $\mathcal{H}_2$ is indiscernible in the world $\mathcal{H}_1$ from $\mathcal{H}_1$ itself, and the world $\mathcal{H}_3$ is indiscernible in the world $\mathcal{H}_2$ from $\mathcal{H}_2$ itself, then the world $\mathcal{H}_3$ must be indiscernible in the world $\mathcal{H}_1$ from $\mathcal{H}_1$ itself. Indiscernible differences between worlds add up and become discernible. Besides, transitivity of R would make every instance of "$\triangle\alpha \supset \triangle\triangle\alpha$" valid. We obviously don't want this as a theorem in our logic of clarity, because it will lead us straight down the slippery slope of sorites reasoning again.

[17] See also [13, p. 272] for a rather technical discussion of the counterpart of Brouwer's axiom in connection with different ways of measuring differences between worlds.

Now, if a sentence follows as a logical consequence from a set of premises which are all clear, it is very reasonable to assume that this sentence must itself be clear, which justifies the following axiom schema:

(AP) $\vdash \triangle(\alpha \supset \beta) \supset (\triangle\alpha \supset \triangle\beta)$.

(I refer to this axiom as "AP" for "the Axiom of clarity Propagation".) Actually, I have already made intuitive appeal to (AP) in informal reasoning about the sorites problem.

In a logic of clarity we also need a way of introducing the clarity operator into an argument in the first place. Now a valid sentence can be regarded as a logical consequence following from the empty set of premises, which by default is a set of clear premises. So the justification of (AP) will also justify the following rule of inference:

(CI) If $\vdash \alpha$ then $\vdash \triangle\alpha$.

(I refer to it as "CI" for "the rule of Clarity Introduction".) I have in fact also appealed to (CI) before, in the same informal argument where (AP) is invoked.

The resulting axiomatization[18] of the logic of clarity is actually a well known system of modal logic (with the only difference that clarity is substituted for necessity), the system T from Kurt Gödel.

Moving on to a quantified logic of clarity and vagueness, we have to add a universe, $\mathcal{U}$, that is, a domain of objects, to the model structure already described. A model of a language, $\mathcal{L}$, of predicate logic which contains the clarity operator will basically consist in an assignment of an extension, a set of n-tuples of objects in $\mathcal{U}$, to every n-place predicate P^n in $\mathcal{L}$ for every draft world $\mathcal{H} \in \mathcal{K}$. The interpretation of open atomic sentences is then given as follows:

> For every $\mathcal{H}$, $\phi(P^n(x_1, \ldots, x_n), \mathcal{H}) = \top$ relative to an assignment of elements in $\mathcal{U}$, $a_1, \ldots, a_n$, to the variables $x_1, \ldots, x_n$ if and only if $\langle a_1, \ldots, a_n \rangle \in \phi(P^n, \mathcal{H})$.

And the interpretation of quantified sentences is determined by the following rule:

(R$\exists$) For every $\mathcal{H}$, $\phi((\exists x)A(x, y_1, \ldots, y_n), \mathcal{H}) = \top$ relative to an assignment of elements in $\mathcal{U}$, $b_1, \ldots, b_n$, to the variables $y_1, \ldots, y_n$ if and only if there is an $a \in \mathcal{U}$ such that $\phi(A(x, y_1, \ldots, y_n), \mathcal{H}) = \top$ relative to an assignment of $a, b_1, \ldots, b_n$ to $x, y_1, \ldots, y_n$.

[18] I am indebted to Williamson, [13, p. 270–272] for the proposed axiomatization.

A discussion of which axioms should be added to a quantified logic of clarity—apart from the axioms of standard predicate logic—must wait until we go into the problem of vague objects and vague reference.

6 The Hermeneutics and Dialectic of Vagueness

Let us now take a closer look at the picture of vagueness that is about to emerge: We have seen that a solution to the sorites problem presupposes that a statement can be vague and even so have a definite truth value, being either true or false. Far from being forced to give up the principle of bivalence, we are in fact compelled to hold fast to it to avoid the absurdities sorites reasoning leads to. By keeping to bivalence, we can even make sense of a logic of vagueness and clarity, which turns out to be a garden variety of standard modal logic. But how plausible is it, really, to claim that statements whose truth or falsity cannot be decided because of a lack of criteria, should even so in general be regarded as having a definite truth value, when there apparently is nothing that can make them true or false?

One reply is that we do not have to make this claim; we can content ourselves with the observation that at least in some cases vague statements do allow of having definite truth values, and leave all the cases where this is not obvious out of consideration. That way we can apparently both solve the sorites problem and make sense of a logic of clarity, so this strategy might be tempting as an easy way out. But it is clearly not a satisfactory answer to the challenge of vagueness.

The epistemic view of vagueness, as expounded and defended by Williamson[19], is more promising. On this view vagueness is a condition of ignorance. With an instructive example of ignorance as to the exact number of people present in a stadium[20], Williamson succeeds in showing clearly the parallel to the problem of deciding exactly how many grains are needed to make up a heap. But there is also a difference here, which I think Williamson tends to play down: In the case of our not being able to know exactly how many people are present in the stadium, in spite of our ability to make a rough estimate, there is no doubt that there really is an exact number, and that this number could be known, at least in principle. In the case of our

[19] See [13, ch. 7–8]. In the preface to his book Williamson makes the confession that he started out with the intention of refuting the epistemic view, but became convinced that the view is correct as a result of his efforts.

[20] [13, §8.2–8.3].

not being able to decide exactly how many grains make up a heap, on the other hand, it looks like there is no way that this number could ever be known, not even in principle; and it is very natural then to entertain certain doubts about the existence of such a number, even though a solution to the sorites problem seems to require that it must exist. A satisfactory account of vagueness ought to be able to remove these doubts.

Like Williamson I too think that vagueness is best understood as a special condition of ignorance—ignorance due to a lack of criteria for deciding, that is. But I also think that his defense of the epistemic view leaves too many questions open to be really satisfactory. In the hope of being able to throw new light on some of the difficulties, I want to probe into the phenomenon of vagueness from a somewhat different angle now.

I will do so by sketching out a kind of hermeneutical theory of the dynamics of linguistic usage and the dialectic of concepts and criteria. In the philosophy of language today it is a widely accepted view that a concept is not, as for instance Kant seemed to think, reducible to a fixed set of criteria, making up a necessary and sufficient condition of its application. We have instead become used to think that the application of a concept in a judgement is governed by more circumstantial evidence of a sort, in the form of a balancing of more or less provisional indications and counterindications that may in themselves be neither necessary nor sufficient conditions.[21] This insight is in itself a concession to the significance of vagueness, and I will therefore take it as my starting point.

As children we acquired our mastery of language only step by step. Our understanding of the concepts expressed by the words the community around us used was limited and partial at first, becoming more and more complete as we were gradually initiated into the linguistic practice. We learned the meaning of words little by little from practical examples. Meaning is not use, but indeed determined by use. From the different circumstances we heard a linguistic expression being used in we gleaned a provisional notion of its sense, and we then tried to use this expression in the same meaning ourselves. At last our apprenticeship was over, and we could take our places as full members of the linguistic community. But that does not mean, as it is easy to think, that the process of learning had come to an end. As I see it, even as fully competent speakers of a language we remain in a sense learners all of our life, because a language is not a static structure, but always in a process of evolution.

[21] Cf. Wittgenstein [14], Quine [9], Putnam [8].

Each application of a linguistic expression in a new situation is more than just a new example of how that expression has been traditionally used; it is strictly speaking also a new interpretation of it in the light of new evidence. In so far as the new application is deemed appropriate it will therefore attain the status of a more or (usually) less important source of future correct usage. This is parallel to the role given to any final court decision as a future legal source. To the extent that a concept is merely an abstract of previous experience it cannot capture the novelty of a new experience, and this means, as I see it, that our concepts must necessarily be open, always allowing of new criteria for their application. Because of that they will also necessarily be vague to a certain degree. A conceptual scheme will always be sketchy and provisional, waiting for new details to be filled in. That does not mean, however, that there are particular cases of vagueness that can never be made clear. On the contrary, I think that any particular case of vagueness is capable of being clarified in the course of linguistic development, even the vagueness of the term "heap" if people will ever become sufficiently interested in removing it: Imagine that having heaps of sand in one's garden becomes a new fad, and—as is not unlikely to happen in that case—a special tax is laid on them. Then it will immediately become a legal question of great interest what is a heap and what is not, and great efforts will be made to find new criteria for a more precise application of that word. It is far from certain that the issuing legal battle will result in anything like justice, but in so far as we can talk about justice in a case like this at all (and I think at least that we would) there must also be a fact of the matter.

In my opinion, an expression is vague only relative to the current stage of the development of a linguistic practice. That the expression is vague today does not imply that it will remain so eternally.[22] There are many examples of terms that have acquired a more accurate meaning in the course of scientific development; the application of the term "water", for instance, has been supplied with new and more precise criteria as a result of the achievements in the field of chemistry of the last 200 years[23]. So on my view of vagueness, that it is unclear because of vagueness whether p means that it is not knowable now because of a lack of criteria whether p or not, not that it will remain unknowable forever. That it is clear that p means correspondingly

[22]And the other way round: What seems clear today may become vague tomorrow.—I am grateful to Aanund Hylland for this observation.

[23]Putnam's twin earth example of the intension and the extension of the word "water" in [8] is instructive in this connecton, apart from the use he puts it to.

that p and that it is not unknowable now because of vagueness whether p. (To say that it is knowable that p would be too strong, for there are other sources of ignorance than vagueness.)

I am actually making three claims here: first, that new criteria can be added to our former criteria for applying a certain concept, or even replace some of them, with the result that what was vague before now becomes clear; second, that this can occur in such a way that the new criteria are introduced with a justification and are not chosen arbitrarily; and third, that when a change of criteria is carried through in this way and no stipulation is involved it will not lead to a change of the concept in question.

I think the truth of the first claim is rather obvious: this happens all the time in the process of scientific development. The truth of the second claim is perhaps not so obvious, but again, scientific progress does not consist in making arbitrary terminological choices, but in discovering more of reality[24]. This is really a partial justification of the third claim as well, because we cannot say, for instance, that the science of chemistry has given us a new concept of water, only that it has taught us more about the nature of water so that we have now better criteria for deciding what falls under this concept and what does not. But there is an additional difficulty here, vagueness may conceal an ambiguity that only becomes clear when new and more precise criteria are introduced. For instance, it came as something of a surprise when it was discovered that the word "jade" was actually used ambiguously to denote two different substances.[25] However, we should not let ambiguity bother us too much. Different interpretations of an ambiguous statement may differ in truth value, but given a particular interpretation the statement will be either true or false. Ambiguity is no threat to the principle of bivalence.

I do not claim that every case of vagueness will be made clear as a result of future epistemic progress, only that there is no case of vagueness that might not in principle be made clearer this way. And, of course, I don't regard the suggestions I have made here as anything like a conclusive argument for the correctness of the picture of vagueness I have been sketching; I just want to recommend this view for consideration.

Seeing vagueness and clarification in the perspective of conceptual maturation might even inspire one to attempt an inquiry into the dynamics of epistemic evolution as an interplay between linguistic devolopment and the

[24]The issue of natural kinds is highly relevant here.

[25]See again [8].

growth of factual knowledge. This could be done in many ways. If we think not only that language is vague, but that reality itself must be vague because of the inherent vagueness of every conceptual scheme, a picture of a world in the making will emerge, a fluid draft world in the process of continuous creation, becoming more and more finished and complete due to constant conceptual revisions and amendments. I am not going to follow up these speculations, but I think that Hegel had something like this in mind when he worked out what he called a system of logic[26]. At least, one of the key concepts in Hegelian dialectic is directly related to the sorites problem: that of a qualitative leap, quantity issuing into quality.

7 Vague Objects or Vague Reference? Vagueness *de re* and *de dicto*

Is reality vague? Are the objects we talk about underdefined and because of that fuzzy? Or is it just vague which objects we actually refer to?

This is in itself a vague issue. A lot of precisification is needed before we can have any hope of giving an answer to these questions. At first sight it may appear that we have a choice between two incompatible views here, but that is not the case, as we shall see. On reflection one might come to think the opposite, that there is no real difference between the two alternatives, but that is wrong too. The truth, as I see it, is that they complement each other: When we look at vagueness in the relationship between word and object from an internal point of view, that is from within the same language as the words belong to, it will appear that the objects we talk about are sometimes vague, but we still have to take into account that it will also be vague at times what a term refers to or is true of. When we look at vagueness from an external viewpoint, from the vantage point of a metalanguage, it will appear that it is primarily the reference or extension of the terms of the object language which is vague, but that does not exclude that the ontology of the metalanguage may be vague as well, and the objects we refer to in the metalanguage will in fact sometimes be vague in the sense that it is unclear how they relate to the terms of the object language.

Our first task will be to make clear sense of the notion of a vague object. I shall start by proposing the following definition: An object is vague if and only if for some property it is unclear whether the object has this property or not.

[26] See [5] and also [4].

To elaborate on this I shall make use of the distinction between *de re* and *de dicto* modalities. As an example, consider the two ways of understanding the statement that the positive square root of 81 is necessarily greater than 7; this statement is really ambiguous between a *de re* and a *de dicto* sense. On the *de re* interpretation, the statement says, truly, that it holds of the number that is the positive square root of 81 that it is necessarily greater than 7. Writing "$P!$" for "is the positive square root of 81"[27] and "G" for "is greater than 7", we can paraphrase this as

(1) $(\exists x)(P!x \,\&\, \Box Gx)$,

with the modal operator in the scope of the existential quantifier. On the *de dicto* interpretation it says, truly again, that there is necessarily a number which is both the positive square root of 81 and greater than 7. We can paraphrase this as

(2) $\Box(\exists x)(P!x \,\&\, Gx)$,

with the existential quantifier in the scope of the modal operator. Taken in the *de re* sense, the statement is true regardless of how we refer to the number in question, whether as "the positive square root of 81", as "9", or as "the number of planets". Taken in the *de dicto* sense, however, substituting a coreferential term for "the positive square root of 81" may change the truth value of the statement; for example, it is presumably false that there is necessarily a number which is both the number of planets and greater than 7. From (1), but not from (2), it can be inferred that

(3) $(\exists x)\Box Gx$,

which has the meaning that there is an object that has the secondary, modal property of necessarily having the primary property of being greater than 7.

Applied to an example of vagueness, there will apparently be a corresponding ambiguity between a *de re* and a *de dicto* sense of the statement that it is unclear whether the longest river in Norway is exactly 611073 meters long. This can be taken to mean, *de re*, that it holds of the longest river in Norway (Glomma) that it is unclear whether it is exactly 611073 meters long, or *de dicto*, that it is unclear whether there is a river in Norway that is longer than any other and which is exactly 611073 meters long. Writing

[27] I follow the convention of writing "$P!x$" as short for "$(\forall y)(Py \equiv (x = y))$", where "$P$" in this case must be taken to mean "is a positive square root of 81".

“$N!$” for “is the longest river in Norway” and “L” for “is exactly 611073 meters long”, we get the following two paraphrases, *de re*:

(1′) $(\exists x)(N!x \,\&\, \nabla Lx)$,

and *de dicto*:

(2′) $\nabla(\exists x)(N!x \,\&\, Lx)$.

Again, from (1′), but not from (2′), it can be inferred that

(3′) $(\exists x)\nabla Lx$,

which has the meaning that there is an object that has the secondary, vagueness property of being such that it is unclear whether it has the primary property of being exactly 611073 meters long. Keeping to the definition I have already given, I shall now say that an object is vague if and only if it has a secondary (or higher-level) property of this kind, a vagueness property. The question of whether there are vague objects will then become a question of whether it really makes sense to apply the *de re*/*de dicto* distinction to cases of vagueness and clarity, and, if so, whether it can ever be truly asserted *de re* about some object, a, that for some property, P, it is unclear whether a has P or not.[28]

That something—a modal condition in the widest sense—is true of an object *de re* ought to mean that it holds true no matter how the object is referred to. This is my understanding of the *de re* concept, and this is how I have presented it here. But, unfortunaly, this concept has been tampered with so much in the course of the last three decades' work in the field of direct reference theory, particularly in connection with the controversy over the notion of belief *de re*, that it is scarcely recognizable anymore. Today people talk freely about different *de re* ways of referring to an object, which to me makes no sense at all. Williamson refrains from considering the possibilities of a quantified logic of clarity and vagueness because of the alleged difficulties with the *de re* concept[29]. I think that is a pity; and what he achieves in this field is consequently of little value, as I see it[30]. I find his discussion of vague objects and vague identity both confused and confusing, and I am not going to follow him.

[28] I first presented this concept of vague objects in my [2].

[29] [13, p. 270].

[30] See [13, ch. 9].

I shall instead keep to what I see as the original and authentic *de re/de dicto* distinction, and my question will be whether objects can be vague in the sense that vagueness properties can be truly ascribed to them.

Could the statement that it is unclear whether the longest river in Norway is exactly 611073 meters long be meaningfully taken in the *de re* sense as an example of a true vagueness property ascription? I think so. The statement is probably true, and the vagueness of the situation it describes seems not to be due to the way the river Glomma is referred to; what it claims to be unclear will not become any clearer if we refer to this river in a different way. For example, to use the name of the river would not clarify anything in this case, for it still seems to be true that it is unclear whether Glomma is exactly 611073 meters long. It would in particular not be of any help to try to refer to Glomma with the definite description "the river in Norway which is exactly 611073 meters long", for it is not even clear that we can succeed in referring to anything with this description. Because of that the statement will still be true if the reference succeeds; in other words, if it is not, this can only be due to reference failure. But if the way Glomma is referred to in the statement is not important to the question of its truth so long as the reference is successful, must not that mean that there is a river which in itself is such that it is unclear of it whether it is exactly 611073 meters long, a river which as an object of discourse has so far not be given a definition that is precise enough to make possible an exact answer to the question of how long it is? I find it at least very tempting to think that this is so, with the consequence that Glomma must be a vague object according to my definition.

It is perhaps in place here to say a few words about the relevance of the distinction between genuine and nongenuine singular terms and of the related distinction between rigid and nonrigid designators to the topic we are discussing. A genuine singular term is one that, like a variable, does not in any way describe (or ascribe something to) the object it refers to, and if it is a constant it is also supposed to show the same logical behavior as a variable, particularly in permitting unrestricted existential generalization: If "a" is a genuine singular constant, the inference from a statement of the form "Fa" to "$(\exists x)Fx$" should always be valid, also when "F" expresses a secondary property, excluding in this case a separate *de dicto* interpretation of the statement from the *de re* one. It is controversial whether genuine singular constants exist, however, and, if so, how their reference is fixed, but I shall not go into that; I will just mention that proper names, like "Glomma", as used by competent speakers are among the candidates for the status of being genuine singular constants on most theories of direct reference. A rigid designator is really something else; Kripke[31] defines it as a singular term that keeps to the same referent in all possible worlds, unlike nonrigid designators, typically definite descriptions, whose reference will typically vary from one possible world to the next. Genuine singular terms will be rigid designators, but rigid designators are not necessarily genuine singular terms. “The prime number between 14 and 18” is an example of a rigid designator that is not a genuine singular term. However, all rigid designators can be considered to behave as genuine singular terms in modal contexts, for the *de dicto* interpretation of a modal construction on a rigid designator always implies the *de re* interpretation of it.

In connection with the logic of clarity it is a distinction between precise and vague designators, parallel to but not coinciding with the distinction between the rigid and nonrigid designators of modal logic, which is primarily of interest: A precise designator is a singular term that refers to the same object in all draft worlds which can be reached from the real world by a chain of indiscernibility relation links, while a vague designator typically shifts its reference from one such draft world to the other.

Seen in the internal perspective, from within the language they belong to, genuine singular constants (in so far as they exist) will appear as precise designators. In this perspective rigid designators will in general appear as precise ones as well, while nonrigid designators (most definite descriptions) may either be precise or vague according to the circumstances. Seen in the external perspective of a metalanguage, neither rigid designators in general nor the special case of genuine singular constants will as such appear as precise designators.

8 Must Vague Objects Have a Vague Identity?

The concept of a vague object that comes out of these considerations—an object which has so far only been provisionally outlined and awaits future completion by more exact definition—is different from the concept of vague objects as indeterminate entities which is commonly being discussed. This difference is not surprising, considering that vague statements have usually been regarded as being neither true nor false, while I have kept to the principle of bivalence in my exploration of vagueness.

[31] [7].

Discussions of vague objects have often turned on the question of vagueness and identity, and those who defend vague objects also tend to think that vague objects must have a vague identity. In this field, Evans's brief note on the problem[32], arguing that objects with a vague identity are impossible, along with Salmon's similar, independent argument for the same conclusion[33], has stirred a lot of controversy. Evans apparently thinks—like his opponents—that having a vague identity is the decisive mark of a vague object. He also takes a vague statement to be of indeterminate truth value, reading "$\triangle$" as "Definitely" and "∇" as "Indefinitely", which creates some additional problems that in my opinion have only contributed to leading the debate astray.

His argument goes like this: As a hypothesis to be refuted, suppose that it is unclear, or indefinite, whether a and b are the same object:

(1) $\nabla(a = b)$.

This implies that b has the property of being such that it is unclear, or indefinite, whether it is identical with a:

(2) $\hat{x}[\nabla(x = a)]b$.

But obviously, it is not unclear, or indefinite, whether a is identical with itself:

(3) $\sim\nabla(a = a)$,

with the consequence that a does not have the property of being such that it is unclear, or indefinite, whether it is identical with a:

(4) $\sim\hat{x}[\nabla(x = a)]a$.

Then a lacks a property that b has, and it follows by Leibniz's law that a and b are different objects:

(5) $\sim(a = b)$.

But then it is not unclear whether $a = b$. This is a *reductio ad absurdum* of the idea that an object can have a vague identity. Evans apparently thinks, unwarrantedly, that it also shows that the notion of a vague object must be incoherent. But is the body of the argument, the inference from the premises (1) and (3) to (5), valid?

[32] [3].
[33] [12].

With my concept of vague objects, I have no problems at all with accepting Evans's argument, in spite of the fact that there are certain difficulties with his understanding of the "∇" and "$\triangle$" operators and their relationship, and also with the implications he thinks the argument should have. The conclusion that the idea of vague identity is absurd is in full accordance with the insight, central to modal logic, that identity is a relation that obtains necessarily if it obtains at all, and we need not speculate, as Evans does, that if we suppose that the logic of vagueness and clarity viewed as a modal logic is as strong as S5[34]—which it obviously cannot be, since the power of S5 would plunge us straight down the slippery slope of sorites reasoning—then "(1)—(4) and, presumably, Leibniz's law, may each be strengthened with a 'Definitely' prefix, enabling us to derive

(5′) $\triangle{\sim}(a = b)$"

—this will follow anyway. In a quantified logic of clarity with identity, the universal closure of "$(x = y) \supset \triangle(x = y)$" ought to be considered valid[35], and no instance of "$\nabla(\zeta = \eta)$", equivalent to "${\sim}\triangle(\zeta = \eta)\,\&\,{\sim}\triangle{\sim}(\zeta = \eta)$", will ever be true. The impossibility of vague identity is not an argument against vague objects, however. Let an object be as vague as it may, it has nevertheless a clear identity as the object which is exactly as fuzzy as it is. What people think of as problems in this connection have more to do with vague designators than with vague objects, as I see it.

I am convinced that Evans intended "a" and "b" to be understood as genuine singular terms, either variables or constants. At any rate, his argument presupposes that the two terms must be precise designators; otherwise, on a *de dicto* reading of (1) and (3), the inference from (1) to (2) and the inference from (3) to (4) would both be invalid.

Let me take the puzzle of Theseus' ship as an example of what could be considered a vague identity: It is not clear, as we recall, that the ship Theseus sailed home from Crete, and the ship that returned from Delos shortly before the death of Socrates, were the same.

What we should say about this case, and of all other cases similar to it, is that the two definite descriptions in question, in our case "the ship Theseus sailed home from Crete" and "the ship that returned from Delos shortly before the death of Socrates", cannot both be precise designators—not that

[34] Distinguished by the axiom "$\Diamond\Box p \supset \Box p$".

[35] But see below, in the discussion that concludes my model-theoretic exposition of underdefined objects, for a qualification of this claim.

the identity of the object or objects the two descriptions refer to is vague. If we claim that both are precise designators, the identity statement "The ship Theseus sailed home from Crete = the ship that returned from Delos shortly before the death of Socrates" must be taken to be vague *de re*, and we will have to maintain that it is unclear of the objects denoted by the two descriptions whether they are the same object or not; I think that Evans's argument should at least be sufficient to show that this is ill-advised. So we ought instead to assume that at least one of the two definite descriptions is a vague designator; then the identity statement will only be vague *de dicto*, that is, the vagueness will be *de dicto* with respect to at least one of the two descriptions.

Let us now see what the outcome will be if one of the two definite descriptions is a precise designator. Actually, (the Greek translation of) the description "the ship that returned from Delos shortly before the death of Socrates", ought to have been a precise designator in the usage of the contemporaries of Socrates who survived him, for to them it would be no problem to identify a ship under this description[36]. It seems that the description as used by them must then denote an object that is vague in my sense: from their standpoint it would be unclear of the ship that returned from Delos shortly before the death of Socrates whether Theseus sailed it home from Crete more than 800 years earlier. It follows from this, granted that Evans's argument is valid, that the Athenians of the early 4. century B.C. could only use the other description, "the ship Theseus sailed home from Crete", as a vague designator, for it would be impossible for them because of the vagueness to identify a ship under that description. To appeal to the name of the ship would not help them. Let us assume that Theseus had dubbed his ship "Pistis", and that the ship that returned from Delos many centuries later was still called "Pistis"; what would then be unclear to the contemporaries of Socrates was whether Pistis was the same ship as the ship called "Pistis" by Theseus and his contemporaries. Because of the vagueness involved, the contemporaries of Socrates simply could not use the name "Pistis" to refer to what would clearly be the same ship as the one Theseus referred to by that name, and in their usage "the ship Theseus called 'Pistis'" would therefore be a vague designator.

Admittedly, there is more to say about the question of vague identity statements and the objects they concern than I have done so far, and once

[36]Correspondingly, the description "The ship Theseus sailed home from Crete" ought to have been a precise designator in the usage of the contemporaries of Theseus.

I have finished my presentation of vague objects as a preparation, we shall go deeper into this issue by way of a model-theoretic consideration.

That an object, a, is vague means, on my conception, that it has at least one vagueness property, that is, that for some predicate, "$F\zeta$", "$\nabla F\zeta$" can be truly ascribed to a, or in other words, that the statement "∇Fa" is true *de re*, thus entailing "$(\exists x)\nabla Fx$". Besides, either "$F\zeta$" or "$\sim F\zeta$" is true of a, so there is no truth value gap.

No restriction is laid on the predicate "$F\zeta$", so it may either be a primary predicate, atomic or composite, containing no clarity or vagueness operator (strictly speaking no modal operator at all, in the widest sense of "modal"), and express a primary property, or it may be a secondary, tertiary or n-ary ($n > 3$) predicate, consisting of one ore more primary predicates and at least one prefix somewhere in the structure that contains one of these operators or an iteration of them, and it will then express a secondary, tertiary or n-ary vagueness or clarity property. Because of this, vague objects allow of degrees of vagueness and clarity. For instance, an object may be such that it is unclear of it whether it has a certain primary property, or such that it is unclear of it whether it is clear that it has this property, or such that it is unclear of it whether it is unclear whether it has the primary property in question, and so on. Still, Leibniz's law holds of such objects: $a = b$ if and only if a and b have all their properties in common, primary properties as well as secondary and higher-level properties of vagueness and clarity.

An interesting question now arises: Can there be two different objects that share all their primary properties and differ only in their secondary or higher-level vagueness and clarity properties? Can there be, for instance, an object a that is different from an object b only in the respect that it is unclear whether a has the property P while it is clear that b has it? (Then a must have P too, but that is not clear.) This might appear to be a weird situation, but even so it is clearly a theoretical possiblity given our conceptual apparatus. So, does this situation ever occur in reality?

To clarify the question, consider again the vagueness of the concept of a table: it is not clear which molecules at the surface of a table are parts of it and which are not. Now this must be taken to mean that a table is a vague object, in so far as we can refer to it with a precise designator. Even so, on my conception of vague objects a table does have clear boundaries, although it is not clear exactly where they are, so that for every molecule at its surface it is either true or false that it is part of the table; it is just that the criteria for deciding this are still lacking.

I am sitting at a table now. It is unclear exactly which aggregate of molecules it comprises. There is a great number of aggregates of molecules in front of me which might each coincide physically with the table I am sitting at, but only one of them actually does. Let us call these aggregates of molecules “tabules”. One and only one tabule coincides physically with the table. This tabule must share all of its constituent molecules with the table, then, but while it is clear which molecules are part of the tabule, it is unclear for some molecules whether they are part of the table or not.

This could make us think that the table is a vaguer version of one of the tabules; we could easily take it to be an example of a situation where two objects differ only in degree of vagueness, in the respect that one of them is better defined than the other; but it is not. For I am only sitting at one table, not at two. Because of that the tabule cannot be a table, since it is a different object from the table I am sitting at, better defined in a certain respect. The property of being a table is presumably a primary property, vague, but not a vagueness property; so the table and the tabule differ also in the respect that one of them is a table while the other is not.

It seems that the relationship between geographical areas viewed as objects makes up a much better case for what could be considered situations where one object differs from another only in its vagueness and clarity properties. Unlike tables, areas contain, are contained in, and overlap other areas. Take the Oslo area as an example: It is in no way clear what exactly it comprises. It is not clear, for instance, that it does not coincide with the city of Oslo and the county of Akershus. Let us suppose that it does. The geographical region that consists of Oslo and Akershus is much better defined, however, so it seems that the Oslo area can then be regarded as an object that is different from the region of Oslo and Akershus only in being vaguer. On the other hand, one could also contest that the Oslo area really is a vague object, maintaining instead that in a case like this it is the designation, “the Oslo area”, that must be considered to be vague.

9 Underdefined Objects

Now the time has come to use our apparatus of model-theoretic semantics to throw more light on the issue of vague objects and identity. Let us start with the river Glomma as an example of a vague object in my sense, an underdefined object: it is unclear of Glomma, because of a lack of criteria for deciding, whether it is exactly 611073 meters long.

The proper name “Glomma” is obviously a precise designator. There is no unclarity, neither first-order nor higher-order, as to its reference. It will refer to the same river Glomma in every draft world which can be reached from the real world $\mathcal{G}$ by a chain of indiscernibility relation links, that is, in every draft world $\mathcal{H}$ which is an element in the set $\mathcal{K}$.[37] Let us now assume that in some draft world, $\mathcal{H}$, (which may be the real world) it is true that Glomma is exactly 611073 meters long, but that it is not true in $\mathcal{H}$ that it is clear that Glomma is exactly 611073 meters long. What this means is that there is at least one draft world $\mathcal{H}'$, indiscernible in $\mathcal{H}$ from $\mathcal{H}$ itself ($\mathcal{H} R \mathcal{H}'$), where it is not true that Glomma is exactly 611073 meters long. In addition we shall assume that it is true in $\mathcal{H}$ that Glomma is clearly a river, and also that Glomma is clearly the longest river in Norway, so in every draft world $\mathcal{H}'$ such that $\mathcal{H} R \mathcal{H}'$, it holds true both that Glomma is a river and that Glomma is the longest river in Norway. We can safely assume so much because a stronger assumption seems also to be warranted, namely that this will hold true in every draft world $\mathcal{H}'' \in \mathcal{K}$, for there is no vagueness involved in these issues, neither first-order nor higher-order. The definite description "the longest river in Norway" will then be a precise designator, whereas the definite description "the river in Norway which is exactly 611073 meters long" will only be a vague designator, for according to our picture of the situation there will be some draft world $\mathcal{H}'$, indiscernible in $\mathcal{H}$ from $\mathcal{H}$ itself, where "the river in Norway which is exactly 611073 meters long" does not denote the river Glomma. (Actually, in this particular case the description will not denote anything at all in a draft world $\mathcal{H}'$ such that $\mathcal{H} R \mathcal{H}'$ where Glomma is not exactly 611073 meters long, but that is another matter.)

We can change the example slightly, and this time assume that it is not true in $\mathcal{H}$ that Glomma is exactly 611073 meters long, while this is to hold true instead in some other draft world, $\mathcal{H}'$, which is indiscernible in $\mathcal{H}$ from $\mathcal{H}$ itself. If these are the only changes, it will still be the case that the description "the longest river in Norway" will be a precise designator in $\mathcal{H}$, whereas the description "the river in Norway that is exactly 611073 meters long" will be a vague designator.

Let us then take a new look at Theseus' ship. Now it will appear that the point of view we choose to see it from matters, and this is what the problem of vague objects in connection with vague identity statements is really about, I think: If we take the viewpoint of the contemporaries of

[37]Note that in a semantics for the logic of clarity and vagueness there is no room for a distinction between being true *in* and true *at* a world—the same vague language $\mathcal{L}$ is spoken in every draft world in $\mathcal{K}$, including, of course, the real world, $\mathcal{G}$, itself.

Socrates who survived him, the proper name "Pistis" will appear a precise designator in their language $\mathcal{L}_1$, denoting the same element in $\mathcal{U}_1$, a_1, in every draft world $\mathcal{H} \in \mathcal{K}_1$, and the statement, (I) "It is clear that Pistis is the ship that returned from Delos shortly before the death of Socrates", will be true, not only in the real world, but, because no vagueness is involved, in every other draft world in $\mathcal{K}_1$ as well, while the statement, (II) "It is clear that Pistis is the ship Theseus sailed home from Crete", will be false. This renders the definite description, (α) "the ship that returned from Delos shortly before the death of Socrates", a precise designator, and the definite description, (β) "the ship Theseus sailed home from Crete", will be a vague designator. If, instead, we choose the standpoint of the contemporaries of Theseus, the proper name "Pistis" will again appear a precise designator, in the language $\mathcal{L}_2$ this time, denoting the same element in $\mathcal{U}_2$, a_2, in every draft world $\mathcal{H} \in \mathcal{K}_2$, but the statement (I) will now be false and the statement (II) true, not only in the real world, but in every draft world in $\mathcal{K}_2$, rendering the description (β) a precise designator and (α) a vague designator (apart from the fact that the contemporaries of Theseus would not be able to use (α) as a definite description at all). Finally, if we look at this from our own natural vantage point, "Pistis" will not appear a precise designator; strictly speaking, the proper name "Pistis" does not belong to our language, $\mathcal{L}_3$; instead $\mathcal{L}_3$ will contain the two vague designators, (P1) "the ship the contemporaries of Socrates called 'Pistis'", and (P2) "the ship the contemporaries of Theseus called 'Pistis'". In some draft world $\mathcal{H} \in \mathcal{K}_3$ (P1) and (P2) will denote different elements in $\mathcal{U}_3$, a_3 and b_3; in some other draft world $\mathcal{H}' \in \mathcal{K}_3$ (P1) and (P2) will denote the same element in $\mathcal{U}_3$, be it a_3, b_3, or a third element, c_3. On this model structure the statements (I) and (II) will both be false in a realistic model, and neither of the definite descriptions (α) and (β) will be a precise designator.

The import of this is that the set of draft worlds, $\mathcal{K}$, and the universe, $\mathcal{U}$, of objects they contain will be dependent on the conceptual scheme of the language, $\mathcal{L}$, in question, with the consequence that talking about vague objects may now be seen as a concession to Kantian idealism. However, as I suggested in the introduction, we cannot expect reality to classify itself independent of a classification system, and even if there is an ultimate language in which the world declares itself to itself, it will still be a language. So let us continue the line of thought we have begun.

An additional difficulty then arises: Say that in a draft world, $\mathcal{H}_1$, an object, a, is the ship Theseus sailed home from Crete, and a different object, b, is the ship that returned from Delos shortly before the death of Socrates, while in another draft world, $\mathcal{H}_2$, a is both the ship Theseus sailed home from Crete and the ship that returned from Delos shortly before the death of Socrates, rendering the object b idle. Then, since b has no individuating primary property in $\mathcal{H}_2$, it seems that the object is misplaced in this world. This is what I see as the real problem in connection with supposed vague identity. How are we to deal with idle objects?

The most natural reaction is perhaps to say that in a case like this, b does not exist in the world $\mathcal{H}_2$—I shall return to that. But first I want to consider a different approach to idle objects: Since we have already acknowledged the possibility that two objects may differ only in their secondary and higher-level vagueness and clarity properties, could not that be the solution to the problem we are facing? Even though b is not individuated by any primary property in $\mathcal{H}_2$, sharing all of its mostly negative primary properties with every other object that is idle in this world, could not different secondary or higher-level properties be sufficient to distinguish between b and other idle objects?

The answer to this question is clearly Yes if the world $\mathcal{H}_1$ is indiscernible in the world $\mathcal{H}_2$ from $\mathcal{H}_2$ itself, for in that case it is true of b in $\mathcal{H}_2$ that it is unclear whether it is the ship that returned from Delos shortly before the death of Socrates, and this will be enough to distinguish b from any other object that is idle in $\mathcal{H}_2$. But if $\mathcal{H}_2$ does not bear the indiscernibility relation to $\mathcal{H}_1$, we still have a problem. Then it is far from obvious that there really is a world $\mathcal{H}'$, indiscernible in $\mathcal{H}_2$ from $\mathcal{H}_2$ itself, where b is individuated by a property it does not share with any other object. (This could be a primary property in $\mathcal{H}'$, or it could be a secondary or higher-level vagueness property, so that b is individuated in $\mathcal{H}'$ by being individuated in a world $\mathcal{H}''$ which $\mathcal{H}'$ bears the indiscernibility relation to.)

The source of this problem is that we have not made the assumption that the indiscernibility relation, R, should be symmetric. On this assumption the problem will dissolve immediately, for then, if $\mathcal{H}_1 R \mathcal{H}_2$, also $\mathcal{H}_2 R \mathcal{H}_1$. However, total symmetry, corresponding to Brouwer's axiom, is more than we really need. We only have to assume that there must be, for every object x, a predicate F such that $Fx \supset \triangle{\sim}\triangle{\sim}Fx$. But this cannot be expressed in first-order predicate logic. Apart from the question of symmetry, the counterpart of the so-called Barcan formula,

(B) $(\forall x)\triangle Fx \supset \triangle(\forall x)Fx$,

and its converse,

(C) $\triangle(\forall x)Fx \supset (\forall x)\triangle Fx$,

will both hold on this approach to the problem of idle objects.

These formulas will not be valid if we choose the alternative: to regard an object as nonexistent in a world where it is not individuated by any primary property. For then we will have to give up the assumption of one domain of objects, the universe, $\mathcal{U}$, which is shared by all draft worlds in $\mathcal{K}$. We have instead to split the universe up in more or less different domains for the different worlds that are elements of $\mathcal{K}$: For every $\mathcal{H} \in \mathcal{K}$, $D(\mathcal{H})$ will be a subset of $\mathcal{U}$.

A further, more serious consequence is that clarity will be separated from vagueness: So far we have assumed that the sentence operators "$\triangle$" and "∇" are tied together by the following equivalences,

$$\triangle p \equiv p \,\&\, {\sim}\nabla p$$

and

$$\nabla p \equiv {\sim}\triangle p \,\&\, {\sim}\triangle{\sim}p.$$

Now we will also have to give up this assumption, because under the present approach a model, ϕ, will only be a partial function, so that $\phi(\alpha, \mathcal{H})$ may be neither $\top$ nor $\bot$ for some pairs of a sentence, α, and a draft world, $\mathcal{H}$. This is not a threat to the principle of bivalence, however, for the statements that lack a definite truth value in a certain world, $\mathcal{H}$, are devoid of meaning in $\mathcal{H}$ because they refer to objects that do not exist in that world.[38] But it has the consequence that it may not be clear that p and not clear that not p, and at the same time not unclear whether p, and this is rather counterintuitive. Here are two examples:

First, the universal closure of "$(x = y) \supset \triangle(x = y)$" will not be valid on this approach, whereas "$(x = y) \supset {\sim}\nabla(x = y)$" will still be valid. This is easy to show: Let "x" and "y" both refer to the object a. Then the identity statement "$(x = y)$" is true. Let us further assume that there is a draft world, indiscernible in the real world from the real world itself, where a is idle, and hence nonexistent, as the present approach will have it. Then the identity statement has no truth value in this draft world, and because of that "$\triangle(x = y)$" is false. But "${\sim}\nabla(x = y)$" is true, for there is no draft world where the identity statement is false.

Second, consider the following scenario: In a world, $\mathcal{H}$, a is the ship Theseus sailed home from Crete, and in every world that is indiscernible in

[38] See Kripke [6] and my [1, p. 122–124].

$\mathcal{H}$ from $\mathcal{H}$ itself a is either, like in $\mathcal{H}$, the ship Theseus sailed home from Crete, or a is idle, and therefore nonexistent on the present approach. (We shall assume that there are draft worlds of both kinds.) Then it will be false, as it intuitively should be, that a is clearly the ship Theseus sailed home from Crete, but it will be true, counterintuitively, that it is not unclear whether a is the ship Theseus sailed home from Crete.

I think it will be difficult to make sense of the separation between the sentence operators "$\triangle$" and "∇" in such a way that both of the examples we have been considering here can be reconciled with our intuitions about vagueness and clarity. Because of that it is perhaps better to choose the alternative strategy and let objects be individuated by secondary and higher-level vagueness properties in worlds where they are idle.

10 Vague Reference

We have so far been looking at the relationship between a vague language and the reality it describes only from the inside, from a standpoint within the same language. In this perspective it appears that objects are sometimes vague in the sense of being underdefined, but then it will also be vague what certain terms denote or are true of. Now we shall take a look at the relationship between word and object from the outside, from the point of view of a metalanguage. This is not an alternative, but a complement to the inside view.[39]

Seen from the outside, from the vantage point of a different (and more precise) language (which could be one person's English as opposed to the English spoken by somebody else), what will appear to be vague is predominantly what the vague terms of the object language refer to or are true of. To take an example: In the internal perspective it is vague exactly which molecules a table consists of. Tables are underdefined objects in this respect. In the external perspective it is vague exactly what kind of object the word "table" is true of, whether it is a kind of object that coincides physically with one type of aggregate of molecules, or a kind of object that coincides physically with another type. Still, to make sense of vagueness in this perspective, it seems that we will have to quantify into vagueness contexts, as I

[39] This is not the place to discuss what Quine [10] calls "the inscrutability of reference", because that is not a vagueness phenomenon. According to Quine, reference is inscrutable for the reason that terms are incomplete linguistic expressions which only have their meanings fixed relative to other terms in the context of the complete sentences they enter into.

am going to show. So objects will continue to be vague on this approach. It will sometimes be unclear of an object as we refer to it in the metalanguage whether it is also the referent of some singular term in the object language, or, if we have to do with a general term, whether it belongs to the extension of that term.

The distinction between singular and general terms is not important in this connection, and we might just as well do away with it for the time being by treating a singular term, τ, as if it were short for the general term "$= \tau$", which is true of one and only one object, the referent of τ, if it is true of anything at all. Then we only have to take one relation, the relation of being true of, into consideration. Not only terms, but sentences as well, can be taken care of in this way: For a sentence to be true is to be true of everything, and to be false is to be true of nothing.

The kind of model-theoretic semantics we have been using in our exposition of vague objects will still be very useful in a discussion of vague reference. This time the language, $\mathcal{L}$, to be modelled is the metalanguage, and the object language will enter into the universe of discourse, $\mathcal{U}$, which may now be considered to consist of two parts: $\mathcal{D}$, the domain of extralinguistic objects, and $\mathcal{O}$, the expressions of the object language. One expression of the metalanguage $\mathcal{L}$ is of particular interest to us here, the dyadic predicate "$\textit{true of}(\zeta, \eta)$", where the first argument is to be an element in $\mathcal{O}$, the second in $\mathcal{D}$. Besides, we shall assume that where the object language contains one vague term, say SHIP, the metalanguage will contain several more precise terms, "$\textit{ship}_1$", "$\textit{ship}_2$", and so on.

That it is vague what the term SHIP of the object language is true of can now be modelled in the following way: In some draft world, $\mathcal{H}_1$, indiscernible in the real world from the real world itself, it holds true that

$$(\forall x)(\textit{true of}(\textsc{ship}, x) \equiv \textit{ship}_1(x)),$$

while in a different draft world $\mathcal{H}_2$, also indiscernible in the real world from the real world itself[40] it holds true that

$$(\forall x)(\textit{true of}(\textsc{ship}, x) \equiv \textit{ship}_2(x)).$$

Then, in so far as the two terms of the metalanguage, "$\textit{ship}_1$" and "$\textit{ship}_2$", are not coextensional, it must be true (in the real world) that

VR $(\exists x)\nabla \textit{true of}(\textsc{ship}, x)$.

[40] Either $\mathcal{H}_1$ or $\mathcal{H}_2$ could be the real world.

VR (for "Vague Reference") is a construal of the statement that it is vague what the term SHIP is true of which I find very plausible, but on this construal objects will still be vague, albeit in a very modest way, in my sense of being underdefined, since VR is expressed by quantifying into a vagueness context.

11 Concluding Remarks

How could the legitimacy and meaningfulness of quantifying into vagueness contexts be contested? I cannot see that there are any good reasons at all for a ban on quantification into contexts governed by the vagueness and clarity operators once we have acknowledged them as sentence operators in the first place, and I also think that I have shown by now how to make good semantical sense of it. But then, seen in the internal perspective of a vague language, the objects we talk about in vague terms will appear to be vague themselves in the moderate sense that it is unclear of them, because of a lack of criteria for deciding, whether they have certain properties or not.

One could attempt to minimize the significance of this by maintaining that the internal perspective with its language dependency distorts our view of reality and creates an illusion of objective vagueness that will dissolve once we choose to look instead at the relationship between word and object from the external standpoint of a metalanguage. But that is not quite true, as we have just seen. First, the metalanguage will be vague too, for there is not, and can never be, a language which is precise in every respect. Second, in order to express the view that it is vague what the terms of a language refer to or are true of, it seems that we will still have to quantify into vagueness contexts. How can we construe this claim so that quantification over extralinguistic objects into a context governed by the vagueness operator is avoided? I can only see one possibility, and that is to reduce it to the rather trivial assertion that for some predicate, P, in the metalanguage, and some term, τ, in the object language, it is unclear whether an object x satisfies P if and only if τ is true of x in the object language. This is trivial, because it amounts to nothing more than claiming that the terms of the object language are sometimes vague (relative to the terms of the metalanguage). On the other hand, I think few people will feel their ontological predilections threatened by having to acknowledge that an object may be vague in the respect that it is unclear of it whether a certain term in some language refers to it or is true of it. Still, language is part of reality, and not an insignificant part. We ought at least to admit so much, even if we should feel reluctant to concede that a conceptual scheme, as embodied in a language, actually conditions the reality we, as human beings, become conscious of.

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