Weak Rough Numbers

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Abstract: A definition of weak rough numbers in weak rough sets is given, the weak rough numbers being a generalization of fuzzy numbers and real numbers in R. Some properties of weak rough interval numbers and weak rough triangular numbers are proved.

Key words: fuzzy sets; rough sets; weak rough sets; weak rough interval numbers; weak rough triangular number

1. Introduction

Rough sets have been introduced by Pawlak in order to describe approximate knowledge of sets. The rough set philosophy's founded on the assumption that with every object of the universe of discourse we associate some information, rough set theory has found many application, it turn out to be a very useful tool for decision support system, especially when vague concepts and uncertain data are involved in the decision process. In some sense the rough sets can be considered a generalization of classical set. Despite of many valuable paper about rough sets have been published, we found very few people discuss the numbers of rough sets. Our objective in this paper is to consider the weak rough interval number, which is the generalization of the fuzzy numbers and real numbers in R.

Despite of fuzzy sets and rough sets are different concepts, there are some connection between fuzzy sets and rough sets. Lotif Zadeh introduced fuzzy set theory in 1965 as away to handle the expression of vague concepts. In Zadeh's theory, a fuzzy subset of X is defined as a function $\mu: X \rightarrow [0,1]$. If A is a fuzzy set subset of X, $\mu_A(x)$ is the degree of x in A, $\mu_A{}^C(x)$ is the degree of x out A, then

$$\mu_A(x) \in [0,1],$$

$$\mu_A{}^C(x) = 1 - \mu_A(x) \in [0,1]$$

We recall that a fuzzy set on X is called a fuzzy singleton if it takes the value 0 for all points x in X except one and a fuzzy number F is a fuzzy set in the real line that satisfies the conditions for normality and convexity.

2. Properties of weak rough points

Now, we recall the definition of rough sets. Let U be any nonempty set and let B be a complete subalgebra of the Boolean algebra T(U) of subsets of U. The pair (U,B) is called a rough universe. Let V=(U,B) be a given fixed rough universe. Let R be the relation defined as followings:

$$A = (A_L, A_U) \in R \text{ if and only if } A_L, A_U \in B, A_L \subseteq A_U$$

The elements of R are called rough sets and the elements of B are called exact sets. Let A=(A_L, A_U) and B=(B_L, B_U) be any two rough sets, then

$$A \cup B = (A_L \cup B_L, A_U \cup B_U);$$

$$A \cap B = (A_L \cap B_L, A_U \cap B_U);$$

$$A \subseteq B \text{ if and only if } A \cap B = A.$$

It is easy to see that $A \subseteq B$ if and only $A_L \subseteq B_L$ and $A_U \subseteq B_U$.

Since the vague concepts, in contrast to the precise concepts, cannot be characterized in terms of information about their elements, we assume that any vague concept can be replaced by a pair of precise concepts-called the lower and the upper approximation of the vague concept. Let R be all real numbers, we consider the weak rough sets and weak rough numbers in R.

Definition 1 Weak rough set A is a pair (A_L, A_U) of standard sets in R, the lower approximation $A \in R$ and the upper approximation $A_U \in R$ where $A_L \subseteq A_U$

The meaning of weak rough set is that if a point lies in A_L, we are sure that the point is in the weak rough set. If a point lies in A_U - A_L. then we are unsure whether the point is, or is not, in the weak rough set. If a point lies out side A_U then we are sure that the point is not in the weak rough set.

Let A= A = (A_L, A_U) and B = (B_L, B_U) be any two weak rough sets, then

$$A \cup B = (A_L \cup B_L, A_U \cup B_U);$$

$$A \cap B = (A_L \cap B_L, A_U \cap B_U);$$

$$A \subseteq B \text{ if and only if } A \cap B = A.$$

It is easy to see that $A \subseteq B$ if and only $A_L \subseteq B_L$ and $A_U \subseteq B_U$.

Proposition 1. Let A, B and C be weak rough sets, then

(1) $A \cup B = B \cup A$, commutativity

(2) $A \cap B = B \cap A$ commutativity

(3) $(A \cup B) \cup C = A \cup (B \cup C)$, associativity

(4) $(A \cap B) \cap C = A \cap (B \cap C)$ associativity

(5) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$ distributivity

(6)$A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$ distributivity

Definition 2. Weak rough set number A is a pair (A_L, A_U) of standard sets in R, where

$A_L \subseteq A_U$, and A_L and A_U are subsets in R.

Denition3 Weak rough interval number A is a pair (A_L, A_U) of standard sets in R, where $A_L \subseteq A_U$, ,and A_L and A_U are interval. i. e. A_L =[a-,a+], A_U=[b-,b+], a-⩾b-,a+ ⩾b+

Example. Let x be the age, A=(A_L, A_U)=([35 ,45];[45 ,55]), if some one at the age 40, we sure he is middle aged, if he is at the age 50, we are unsure whether the he is or is not middle aged. If he is at the age 70 then we are sure that the he is not middle aged, so the weak rough interval number A is a vague concept, it means middle aged.

We recall that F=(b- ,a- ,a+ , b+) is a trapezia fuzzy number means if x∈[a-,a+] then μ_F(x)=1; if x∈[b-,a-] then μ_F(x)= (x-.b-)/(a^{-}-b^{-}); if x∈[a+, b+], then μ_F(x)=(b+-x)/(b+- a+); if x∈(-∞,b-)∪ (b +,+∞) then μ_F(x)= 0. It is easy to see that for any trapezia fuzzy number F=(b- ,a- ,a+ , b+). Let A_L =[a-, a+], A_U=[b-,b+] then A=(A_L, A_U) weak rough interval number. So the weak rough interval number is a generalization of trapezia fuzzy numbers in R.

We consider the order ⩽ between the weak rough interval number. Let W be all weak rough interval numbers in R, Let P,Q∈W; P= ([a- , a+];[b- ,b+])

Q =([c- , c+];[d- ,d+]); P⩽Q if and only P⊆Q, ∧ and ∨ are given by:

P∨Q=([min{a-,c-},max{a+, c +}];[min{b- ,d-}; max{b+, d+ }])

P∧Q=([max{a-,c-},min{a+, c +}];[max{b- ,d-}; min{b+, d+ }])

Theorem1. Let W be all weak rough interval numbers in R, then (W, ⩽, ∧ , ∨) is a distributive lattice.

Next, we consider the operator between the weak rough interval numbers.

Definition 4. Let P1, P2 be the weak rough interval number on R, P1=([a- , a+];[b- ,b+]),P2=([c- , c+];[d- ,d+]),and $\lambda \geq 0$ then:

(1) P=(P_L ,P_U)=([e- , e+],[f- ,f+])=P1 +P 2 is defined as:

P_L= [e- , e+] =[a- , a+]+ [c- , c+]=[a^-+c^- , a^++c^+]

P_U =[f- ,f+])= [b- , b+]+ [d- , d+]=[b^-+d^- , b^++d^+]

(2) P= (P_L ,P_U) =([e- , e+],[f- ,f+])=P1 - P 2 is defined as:

P_L= [e- , e+] =[a- , a+]- [c- , c+]=[a^--c^- , a^+-c^+]

P_U =[f- ,f+])= [b- , b+]- [d- , d+]=[b^--d^- , b^+-d^+]

(3) For any$\lambda \geq 0$ the product. P is defined as:

$\lambda P_L=\lambda$ [e- , e+]= λ[a- , a+]=[λ a- , λa+]

$\lambda P_U=\lambda$ [f- , f+]= λ[f- , f+]=[λ f- , λf+]

(4) The product (-1)P1 is defined as:

P_L= [e- , e+] =(-1)[a- , a+]= [-a- , -a+]

P_U =[f- ,f+])= (-1)[b- , b+]= [-b- , -b+]

Property 2. Let W be all weak rough interval numbers in R, then (W;+) is commutative semigroup, and zero is 0=([0,0],[0,0])

Property 3 Let W be all weak rough interval numbers in R, P,Q∈W, then

(1) - (-P) = P;

(2) P⩽Q iff -P⩽ -Q;

(3) If P⩽Q then P+R⩽Q+R;

(4) If P⩽Q then P- R⩽Q-R;

(5) P- Q=0 iff P=Q;

(6) P-Q = P+(-Q);

(7) P-0=P ; 0-P=-P:

Property 4 Let W be all weak rough interval numbers in R, and P, Q, R∈W, then

(1) $(P\vee Q)+R = (P+R) \vee (Q+R)$;

(2) $(P\wedge Q)+R = (P+R) \wedge (Q+R)$;

(3) $(P\vee Q)-R = (P-R) \vee (Q-R)$;

(4) $(P\wedge Q)-R = (P-R) \wedge (Q-R)$;

(5) $R-(P\vee Q) = (R-P) \vee (R-Q)$;

(6) $R-(P\wedge Q) = (R-P) \wedge (R-Q)$:

Property 5 Let W be all weak rough interval numbers in R, and P,Q,R∈W, then

$$(P-Q)-R = P-(Q+R)$$

Definition 2 Let $A=(A_L, A_U)$ be weak rough set, if $A_L =\{x\}$, then we say A is a weak rough point, denoted by wr x.
It is easy to see that for any weak rough set $A=(A_L, A_U)$, we have $A=\cup\{ x \mid x\in A_L\}$

Definition 3 Let $A=(A_L, A_U)$ be weak rough set, $A_L \subseteq R$,$A_U \subseteq R$ if $A_L =\{x\}$, then we say A is a weak rough number

Definition 4 Let Px, Py be the weak rough number on R, $Px =(\{x\},P^x_u)$, $Py =(\{y\},P^y_u)$, and $\lambda \geq 0$ then:

(1) $Px +Py =(\{x+y\},\{a+b \mid a\in P^x_u;b\in P^y_u\}$

(2) $Px -Py =(\{x-y\},\{a-b \mid a\in P^x_u;b\in P^y_u\}$

(3) For any $\lambda \geq 0$ the product λP is defined as:. $\lambda P =(\lambda\{x\}, \lambda a \mid a\in P^x_u)$

(4) the product (-1)Px is defined as: $(-1)Px =(\{(-1)x\}, \{(-1)a \mid a\in P^x_u\}$

Definition 2 Let $A=(A_L, A_U)$ be weak rough set, $A_L \subseteq R$,$A_U \subseteq R$ if $A_L =\{x\}$, and $A_U = [b- , b+]$ hen we say A is a weak rough triangular number, denoted by wrn x. .

We recall that a triangular number F=(a,b,c) means $\mu_F(b)=1$, if $x\in[a, b]$ then μ

$_F(x)=(x-a)/(b-a)$, and if $x\in[b, c]$ then $\mu_F(x)=(c-x)/(c-b)$; if $x\in(-\infty,a)\cup(c,+\infty)$ then $\mu_F(x)=0$. It is easy to see the weak rough triangular number is a generalization of triangular fuzzy numbers in R.

Definition 4. Let Px, Py be the weak rough triangular number on R,

Px = ({x},[b-, b+]), Py = ({y},[d-, d+]), then the distance of Px and P y is defined as $d(Px,Py)=\max\{|x-y|, |b^{-}-d^{-}|, |b^{+}-d^{+}|\}$

The straightforward verification leads to the following conclusions.

Property 4. Let Px, Py, Pz be weak rough triangular numbers, then

(1) d(Px , Py) = d(Py, Pz);

(2) d(P x, Py) = 0 iff Px = Py ;

(3) d(P x, Py) $\leqslant$ d(Px, Pz)+d(Pz, Py);

(4) d(Px, Py) = d(Px +Pz, Py +Pz):

Definition 4. Let Px_n and Px_0 be the weak rough triangular numbers on R, we say Px_n converge to Px_0 if $d(Px_n, Px_0)$ converge to 0 as $n\to\infty$. .

The rough set concept is a new mathematic approach to imprecision, vagueness and uncertainty. Rough set theory has found many applications, by above definitions; we can extend the real numbers and fuzzy numbers theory to the rough sets theory in some sense.

References

[1] Chanas, S. On the interval approximation of a fuzzy number. Fuzzy Sets and Systems 122(2001), no.2, 353-356.

[2] Davvaz, B. Fuzzy sets and probabilistic rough sets. Int. J. Sci. Technol. Univ. Kashan1 (2000), no.1, 23-28.

[3] Iwinski, T. B. Algebraic approach to rough sets. Bull. Polish Acad.Sci. Math. 35(1987), no.9-10, 673-683.

[4] Pawlak, Z. Rough sets, decision algorithms and Bayes' theorem. European J.Oper. Res. 136(2002), no.1, 181-189

[5] Pawlak, Z. Rough sets. Present state and the future. Rough sets |state of the art and perspectives. Found. Comput. Decision Sci. 18(1993), no.3-4,157-166

[6] Pawlak, Z. Rough logic. Bull. Polish Acad. Sci. Tech. Sci. 35(1987), no.5-6,253-258

[7] Pawlak, Z. Rough sets, Internat. J. Inform. Comput. Sci. 11(5)(1982)341-335.

[8] Radzikowska, Anna, M.; Kerre, E. E. A comparative study of fuzzy rough sets. Fuzzy Sets and Systems 126(2002), no.2, 137-155.

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