Uncertain Fuzzy Rough Sets

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Abstract: A definition of the uncertain fuzzy rough sets is given, the uncertain fuzzy rough sets being a generalization of rough sets and fuzzy rough sets. Various properties are proved, which are connected to the operations and properties on the uncertain fuzzy rough sets.

Key words: uncertain fuzzy rough sets; fuzzy sets

1. Introduction

For the most sets in real world, the boundary of a proposition and its negation is always fuzzy, so L. A. Zadeh introduced fuzzy set theory in 1965 as away to handle the expression of vague concepts. Rough sets have been introduced by Pawlak[7] in order to describe approximate knowledge of sets. In some sense the rough sets can be considered a generalization of crisp set. The algebraic approach to rough sets has been given by Iwinski[4] . Fuzzy sets and rough sets are different concepts. Since they are relevant in some applications, it is useful to combine them as argued by Nanda and Majumdar[3]. Rough sets based on a fuzzy equivalence relation are called fuzzy rough sets.

The purpose of this paper is to introduce the concept of uncertain fuzzy rough sets; the uncertain fuzzy rough set is a generalization of rough sets and fuzzy rough sets. In L. A. Zadeh's theory, A fuzzy subset of X is defined as a function: $\mu:X\rightarrow[0,1]$. If A is a fuzzy subset of X, $\mu_A(x)$ is the degree of x in A, $\mu_A^C(x)$ is the degree of x out A, then

$$\mu_A(x) \in[0,1],$$

$$\mu_A^C(x) =1-\mu_A(x) \in[0,1]$$

Let $\mu_A(x)=\alpha$, $\mu_A^C(x) =\beta$, then $\alpha,\beta\in[0,1]$ and $\alpha+\beta=1$.

Let A,B be the fuzzy set then,

(1) A=B if and only if $\mu_A(x)= \mu_B(x)$

(2) $A\subseteq B$ if and only if $\mu_A(x) \leqslant \mu_B(x)$

(3) the union $C = A \cup B$ is defined as

$$\mu_C(x) = \max\{\mu_A(x), \mu_B(x)\}$$

(4)the intersection $D = A \cap B$ of A and B is defined as

$$\mu_D(x) = \min\{\mu_A(x), \mu_B(x)\}$$

(5) the complement A^C of A is defined as

$$\mu_{A^C}(x) = 1 - \mu_A(x)$$

Let U be any nonempty set and let B be a complete subalgebra of the Boolean algebra T(U) of subsets of U. The pair (U, B) is called a rough universe. Let V = (U, B) be a given fixed rough universe. Let R be the relation defined as followings:

$$A = (A_L, A_U) \in R \text{ if and only if } A_L, A_U \in B, A_L \subseteq A_U$$

The elements of R are called rough sets and the elements of B are called exact sets. Let $A = (A_L, A_U)$ and $B = (B_L, B_U)$ be any two rough sets, then

$$A \cup B = (A_L \cup B_L, A_U \cup B_U);$$

$$A \cap B = (A_L \cap B_L, A_U \cap B_U);$$

$$A \subseteq B \text{ if and only if } A \cap B = A.$$

It is easy to see that $A \subseteq B$ if and only $A_L \subseteq B_L$ and $A_U \subseteq B_U$.

2. Properties of uncertain fuzzy rough sets

Now, we are in a position to introduce the concepts of uncertain fuzzy sets and uncertain fuzzy rough sets.

Definition 1. Let X be an ordinary nonvoid set. A is defined as a set of ordered pairs $\{(x, \mu_A(x), \nu_A(x), \eta_A(x)) | x \in X\}$, the three mappings μ_A, ν_A, η_A from X into the unit interval [0,1], A is called uncertain fuzzy set, if $\mu_A(x) + \nu_A(x) + \eta_A(x) = 1$ for all $x \in X$.

Definition 2. Let U be a set and B a Boolean subalgebra of the Boolean algebra of all sublattice. Let X be a rough set, then $X = (X_L, X_U) \subseteq B^2$ with $X_L \subseteq X_U$.

A uncertain fuzzy rough set $A = (A_L, A_U)$ in X is characterised by maps $\mu_{AL}: X_L \to [0,1]$, $\nu_{AL}: X_L \to [0,1]$, $\eta_{AL}: X_L \to [0,1]$ and $\mu_{AU}: X_U \to [0,1]$, $\nu_{AU}: X_U \to [0,1]$, $\eta_{AU}: X_U \to [0,1]$ with the property that

$$\mu_{AL}(x) \leqslant \mu_{AU}(x) \text{ for all } x \in X_U$$

$$\eta_{AL}(x) \leqslant \eta_{AU}(x) \text{ for all } x \in X_U$$

Let $A = (A_L, A_U)$ and $B = (B_L, B_U)$ be any two uncertain fuzzy rough sets, then

(1) $A \subseteq B$ if and only if

$$\mu_{AL}(x) \leqslant \mu_{BL}(x) \text{ for each x in } X_L$$

$$\nu_{AL}(x) \geqslant \nu_{BL}(x) \text{ for each x in } X_L$$

$$\eta_{AL}(x) \leqslant \eta_{BL}(x) \text{ for each x in } X_L$$

$$\mu_{AU}(x) \leqslant \mu_{BU}(x) \text{ for each x in } X_U$$

$$\nu_{AU}(x) \geqslant \nu_{BU}(x) \text{ for each x in } X_U$$

$$\eta_{AU}(x) \leqslant \eta_{BU}(x) \quad \text{for each x in } X_U$$

(2) the union $C = A \cup B$ is defined as

$$\mu_{CL}(x) = \max\{\mu_{AL}(x), \mu_{BL}(x)\} \text{ for all x in } X_L$$

$$\eta_{CL}(x) = \min\{\eta_{AL}(x), \eta_{BL}(x)\} \text{ for all x in } X_L$$

$$\nu_{CL}(x) = 1 - \mu_{CL}(x) - \eta_{CL}(x)$$

$$\mu_{CU}(x) = \max\{\mu_{AU}(x), \mu_{BU}(x)\} \text{ for all x in } X_U$$

$$\eta_{CU}(x) = \min\{\eta_{AU}(x), \eta_{BU}(x)\} \text{ for all x in } X_U$$

$$\nu_{CU}(x) = 1 - \mu_{CU}(x) - \eta_{CU}(x)$$

(3) the intersection $D = A \cap B$ of A and B is defined as

$$\mu_{DL}(x) = \min\{\mu_{AL}(x), \mu_{BL}(x)\} \text{ for all x in } X_L$$

$$\eta_{DL}(x) = \max\{\eta_{AL}(x), \eta_{BL}(x)\} \text{ for all x in } X_L$$

$$\nu_{DL}(x) = 1 - \mu_{DL}(x) - \eta_{DL}(x)$$

$$\mu_{DU}(x) = \min\{\mu_{AU}(x), \mu_{BU}(x)\} \text{ for all x in } X_U$$

$$\eta_{DU}(x) = \max\{\eta_{AU}(x), \eta_{BU}(x)\} \text{ for all x in } X_U$$

$$\nu_{DU}(x) = 1 - \mu_{DU}(x) - \eta_{DU}(x)$$

It is easy to see that for an uncertain fuzzy rough set $A=(A_L; A_U)$, A_L and A_U are uncertain fuzzy sets, so $\mu_{AL}(x) + \nu_{AL}(x) + \eta_{AL}(x) = 1$ and $\mu_{AU}(x) + \nu_{AU}(x) + \eta_{AU}(x) = 1$ for all x.

Next, we list the main properties of uncertain fuzzy rough sets with respect to the operations $\cup$ and $\cap$.

Proposition 1. Let A, B and C be uncertain fuzzy rough sets, then

(1) $A \cup B = A$, $A \cap A = A$

(2) $A \cup B = B \cup A$, $A \cap B = B \cap A$

(3) $(A \cup B) \cup C = A \cup (B \cup C)$, $(A \cap B) \cap C = A \cap (B \cap C)$

(4) $A \cup (A \cap B) = A$, $A \cap (A \cup B) = A$

(5) $A \subseteq A \cup B$, $B \subseteq A \cup B$

(6) $A \cap B \subseteq A$, $A \cap B \subseteq B$

(7) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$

(8) $A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$

(9) $A \subseteq B$ and $C \subseteq D \Rightarrow A \cup C \subseteq B \cup D$;

(10) $A \subseteq B$ and $C \subseteq D \Rightarrow A \cap C \subseteq B \cap D$

We define the complement A^C of A by ordered pair $(A_L^C; A_U^C)$ of member functions.

Definition 4. The complement of A is defined as $A^c = (A_L^C, A_U^c)$, where

$\mu_{A_L^c}(x) = \nu_{A_L}(x)$ for all $x \in X_L$

$\eta_{A_L^c}(x) = \eta_{A_L}(x)$ for all $x \in X_L$

$\mu_{A_U^c}(x) = \nu_{A_U}(x)$ for all $x \in X_U$

$\eta_{A_L^c}(x) = \eta_{A_L}(x)$ for all $x \in X_U$

Proposition 2. Let A and B be uncertain fuzzy rough sets, then

(1) $(A \cup B)^C = A^C \cap B^C$

(2) $(A \cap B)^C = A^C \cup B^C$

The uncertain fuzzy rough sets is the generalization of rough sets and fuzzy rough sets, we hope that the uncertain fuzzy rough sets will be a very useful concept for vague concepts and uncertain data are involved in the look-ahead reasoning of artificial intelligence and decision making of cognitive science.

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